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ESTIMATING UNITED STATES PHILLIPS CURVES WITH EXPECTATIONS CONSISTENT WITH THE STATISTICAL PROCESS OF INFLATION

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ESTIMATING UNITED STATES PHILLIPS CURVES WITH EXPECTATIONS CONSISTENT WITH THE STATISTICAL PROCESS OF INFLATION*

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ABSTRACT

‘Modern’ Phillips curve theories predict inflation is an integrated, or near integrated, process. However, inflation appears bounded above and below in developed economies and so cannot be ‘truly’ integrated and more likely stationary around a shifting mean. If agents believe inflation is integrated as in the ‘modern’ theories then they are making systematic errors concerning the statistical process of inflation. An alternative theory of the Phillips curve is developed that is consistent with the ‘true’ statistical process of inflation. It is demonstrated that United States inflation data is consistent with the alternative theory but not with the existing ‘modern’ theories.

Keywords: Phillips curve, inflation, structural breaks, GARCH, non-stationary data

JEL Classification: C22, C23, E31

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1. INTRODUCTION

A notable shortcoming of the ‘modern’ Friedman-Phelps (F-P) expectations augmented, New Keynesian (NK) and hybrid theories of the Phillips curve is that they predict inflation is an integrated, or very near integrated, statistical process and that this prediction is a direct result of the personal characteristics of the agents in all three models.¹ For example, in the F-P model this prediction is due to the assumption that agents hold adaptive expectations. In the NK model the coefficient on expected inflation is equal to the discount rate of households and firms. The idea that the statistical process of inflation is due to a characteristic of agents and not due to the behaviour of central banks is an anathema to standard monetary theory where central banks set monetary policy in response to shocks to inflation which in turn determines the long-run rate of inflation. Furthermore, as inflation in developed economies appears to have an upper boundary at some moderate rate and a lower boundary around zero it is unlikely inflation can be ‘truly’ integrated.

While inflation appears not to be an integrated process it is likely that inflation is non-stationary.² For example, to argue the converse that inflation is stationary implies, (i) the stance of monetary policy is unchanging leading to a constant mean rate of inflation, (ii) there is only one expected rate and associated long-run rate of inflation implying there is only one short-run Phillips curve, and (iii) the original Phillips (1958) curve did not ‘break-down’ with changes in the expected rate of inflation towards the end of the 1960s. Furthermore, a constant mean rate of inflation implies that all the ‘modern’ theories of the Phillips curve since Friedman (1968) and Phelps (1967) are irrelevant on an empirical level as there has been no change in the expected rate of inflation. Unless we are comfortable with the implications of inflation as a stationary process we need to conclude that inflation is a stationary process around shifting means. This allows for the numerous long-run and

¹ The F-P and hybrid theories predict that the sum of the dynamic inflation terms sum to one. In the NK model the coefficient on expected inflation is the discount rate. However, the empirical NK literature largely ignores this and considers the sum of the dynamic inflation terms to be one. For simplicity of exposition we will assume the sum of the coefficients on the dynamic inflation terms in the NK and hybrid models is one unless otherwise stated. To paraphrase Milton Friedman’s famous quotation from his Wincott Memorial Lecture delivered in London on 16 September 1970, ‘modern’ theories of the Phillips curve argue that ‘inflation is always and everywhere an integrated phenomenon’.

² The term non-stationary in this paper encompasses all statistical processes other than stationary with a constant mean. It therefore includes stationary around a shifting mean.

expected rates of inflation which are a central component of modern theories of the Phillips curve. Furthermore, deviations from any particular mean rate of inflation are partly due to exogenous shocks to inflation and partly due to the response of monetary policy to those shocks.³

One might also question the role that agents play in the modern theories of the Phillips curve. The agents that populate these theories are very sophisticated and very well informed so as to undertake the highly sophisticated optimising behaviour in these models. Therefore it is inconsistent with the sophisticated nature of the agents that they do not understand something as simple as the statistical process of inflation. In particular, a sophisticated agent is unlikely to make the systematic error over the last fifty or more years of predicting that inflation has a constant mean or is an integrated process. Consequently, an attractive characteristic of any model of how agents form inflation expectations would be that agents do not make systematic errors concerning the ‘true’ statistical process of inflation. In particular, the information set of rational agents should include an understanding of the ‘true’ statistical process of inflation and that this set should not include over the past fifty years the mistaken belief that inflation is (i) integrated or (ii) stationary. We therefore propose a model of expected inflation which is consistent with the statistical process of inflation.

The empirical Phillips curve literature also has its shortcomings. First, the literature that seeks to validate the competing modern theories fails to model the heteroscedastic nature of inflation over the past five decades. Graph 1 of United States quarterly inflation for the period March 1960 to December 2010 shows the variance in inflation increasing during the turbulent high inflation years of the 1970s before declining to lower levels following the ‘Volker deflation’ in the early 1980s.⁴ This ‘clustering’ of high and low variance into discrete periods suggests that the variance in inflation may be serially correlated. Since Engle (1982, 1983), a popular way to accommodate the heteroscedastic characteristics of the inflation data

³ Russell (2006, 2011) makes this argument in more detail.

⁴ Inflation is measured in Graph 1 as the quarterly change in the natural logarithm of the gross domestic product (GDP) implicit price deflator at factor cost multiplied by 400 to provide an ‘annualised’ rate of inflation. See Appendix 1 for details of the data used in this paper.

is to estimate one of a wide range of auto-regressive conditional heteroscedastic (ARCH) type models of inflation.⁵

This brings us to the second shortcoming of the empirical Phillips curve literature where most work proceeds under the assumption that inflation is either stationary or integrated. These assumptions are difficult to sustain as argued above. If instead the ‘true’ statistical process of inflation is a stationary process around a shifting mean then assuming inflation is either stationary or integrated will lead to biased estimates of Phillips curves.⁶ Russell (2011) and Russell *et al.* (2011) demonstrate using United States data that not adequately accounting for the shifts in mean inflation leads to severely biased estimates of Phillips curves.

The shifting mean rate of inflation is also evident in Graph 1. These shifts can be identified formally by applying the Bai and Perron (1998) technique to identify multiple breaks in the mean rate of inflation.⁷ Nine breaks in mean are identified in the inflation data implying there are ten inflation ‘regimes’ within which we believe statistically the mean rate of inflation is constant. The identified mean rates of inflation in each regime are shown on Graph 1 as solid thin horizontal lines. On a visual level the technique appears to have identified all the major shifts in mean over the past fifty years.

This paper therefore considers two questions. First, do Phillips curves that model inflation expectations in a way that is consistent with the statistical process of inflation dominate in an empirical sense the existing three ‘modern’ theories of the Phillips curve? And second, are estimates of the ‘modern’ Phillips curves affected in any meaningful way by accounting for (i) shifts in the mean rate of inflation, and (ii) the heteroscedastic nature of inflation?

⁵ In his seminal Nobel Prize winning paper Engle (1982) introduces the ARCH methodology and demonstrates the technique on a model of United Kingdom inflation. There is a wide range of excellent survey articles on the ARCH methodology including those of Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994) and Engle, Focardi and Fabozzi (2008). For applied work see Engle (1982, 1983, 1988), Cosimano and Jansen (1988), Baillie, Chung and Tieslau (1996), Grier and Perry (2000) and Boero, Smoth and Wallis (2008).

⁶ This is a generalisation of the Perron (1989) argument that stationary processes with breaks are easily mistaken for integrated processes.

⁷ See Appendix 2 for details of the Bai-Perron estimated breaks in the mean rate of inflation.

In the next section we briefly consider the ‘three’ modern theories of the Phillips curve so as to identify the source of the prediction that inflation is an integrated, or near integrated, process. Section 3 suggests a fourth theory of the Phillips curve where the formation of expectations is consistent with agents knowing the statistical process of inflation. Section 4 sets out and Section 5 estimates a general hybrid model of inflation that nests all four theories of the Phillips curve allowing for possible ARCH effects and shifting means in the data. Consistent with Castle (2010), Russell (2011), Russell *et al.* (2011) and Nymoen *et al.* (2012), we find that once we account for the shifts in the mean rate of inflation, there is no evidence that expected inflation as commonly measured in the New Keynesian literature plays a significant role in the dynamics of inflation. We also find that the data is inconsistent with the standard interpretations of both the Friedman-Phelps and hybrid models of inflation. In contrast, we find empirical evidence that the Phillips curve proposed in Section 3 that incorporates expectations based on knowledge of the statistical process of inflation is consistent with the data.

2. ‘MODERN’ THEORIES OF THE PHILLIPS CURVE

‘Modern’ theories of the Phillips curve are ‘expectation’ based and incorporate an inflation equation written in general form as:

$$\pi_t = \psi E\{\pi\} + \delta_z z_t \quad (1)$$

where the rate of inflation in period t , π_t , depends on expected inflation, $E(\pi)$, conditioned on available information and a ‘forcing’ variable, z_t .⁸ The latter is measured in a number of ways in the literature including the unemployment rate, the gap between the unemployment rate and some measure of its long-run value, the gap between output and its potential level, real marginal costs, labour’s income share and the markup of prices over unit labour costs. Since Friedman (1968) and Phelps (1967), all three theories believe on an empirical level at least that the ‘correct’ long-run value of ψ is one so that the long-run Phillips curve is ‘vertical’. Leaving aside the different forcing variables, what differentiates these models of

⁸ Equation (1) is general in the sense that the time subscripts associated with the expectations operator, (π) , are ignored as they differ between the three modern theories of the Phillips curve.

inflation is how expected inflation is dealt within the inflation equation. We now briefly describe the F-P, NK and hybrid theories so as to identify what determines the size of ψ in equation (1) in each model.

2.1 *The Friedman-Phelps Phillips Curve*

The expectations augmented Phillips curve of Friedman (1968) and Phelps (1967) assumes adaptive expectations of the form:

$$E_{t-1}\{\pi_t\} = E_{t-2}\{\pi_{t-1}\} + \eta(\pi_{t-1} - E_{t-2}\{\pi_{t-1}\}) \quad (2)$$

where a fixed proportion, η , of the errors in the expectation of inflation are eradicated in each period. Cagan (1956) is the first to incorporate adaptive expectations into estimated models of inflation and refers to η as the ‘coefficient of expectations’ which represents how quickly expected rates of inflation adjust to actual rates of inflation. Backward induction of adaptive expectations implies that expected inflation is a geometrically declining distributed lag of all past rates of inflation such that:

$$E_{t-1}\{\pi_t\} = \sum_{i=1}^{\infty} \eta(1-\eta)^i \pi_{t-i} \quad (3)$$

where $\sum_{i=0}^{\infty} \eta(1-\eta)^i = 1$. Substituting equation (3) into (1) to replace $E(\pi)$ provides;

$$\pi_t = \sum_{i=1}^{\infty} \eta(1-\eta)^i \pi_{t-i} + \delta_z z_t \quad (4)$$

On a practical level equation (4) cannot be estimated with an infinite number of lags. Using the Kyock (1954) transformation we can truncate the number of lags and re-write (4) as:

$$\pi_t = (1-\eta)\pi_{t-1} + \eta\pi_{t-1} + \delta_z z_t = \pi_{t-1} + \delta_z z_t \quad (5)$$

which can be thought of as the F-P Phillips curve. Other possible transformations include that of Almon (1965) and the rational distributed lag function of Lucas and Rapping (1969).⁹ However, to conform with the F-P Phillips curve the lagged inflation terms must sum to one over the truncated number of lags so that there is no trade-off in the long run between the nominal variable, π_t , and the real variable, z_t .

Note three aspects of equation (5). One, the predication that the sum of the dynamic terms equals 1 is a direct result of the assumption of adaptive expectations and not due to any underlying optimising behaviour of the agents in the model. Two, adaptive expectations have a long acknowledged (at least in the rational expectations literature) unappealing implication that agents make systematic errors in their price expectations while they are converging on a new long-run rate of inflation. And three, the desire for the dynamic inflation terms to sum to one is so that long-run money neutrality is maintained in the model.

2.2 *The New Keynesian Phillips Curve*

The N-K Phillips curve responds to two of the perceived short-comings of the F-P Phillips curve by providing optimising microeconomic foundations for the Phillips curve and allowing agents to no longer make systematic errors in their expectations through the use of rational expectations. While there are numerous versions of the NK model, the basic NK model of Gali (2008) provides a general exposition of the NK Phillips curve and allows us to identify the determinants of ψ in equation (1).¹⁰

Gali's basic NK model comprises households and firms. Households undertake inter-temporal optimisation and are price takers in both goods and labour markets. In keeping with a standard classical macroeconomic model, firms also optimise but in contrast with the classical model firms set prices for a differentiated product and follow Calvo (1983) price setting.

⁹ Nerlove (1956) combines the adaptive expectations of Cagan (1956) and the Koyck (1954) transformation to provide the result shown in equation (5). For a clear survey of the econometrics of early distributed lagged models see Griliches (1967).

¹⁰ Gali (2008) sets out a very clear and well-argued basic NK model. The nomenclature used here is the same as that used by Gali where further details and extensions of the model can be found.

In the basic NK model there is a representative infinitely-lived household that seeks to maximise its objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad (6)$$

where C_t is a consumption index and N_t is the hours of employment. The utility function, U , is continuous and twice differentiable, and β^t is the discount rate in period t that households apply to future consumption and employment decisions. Assuming there is a continuum of goods represented by the interval $[0, 1]$ the household budget constraint in period t can be represented as:

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \quad (7)$$

where $P_t(i)$ is the price of the differentiated goods represented by the interval $[0, 1]$, W_t is the nominal wage rate, B_t is the quantity of riskless discount bonds that are purchased in period t and mature with price Q_t , in $t+1$ and T_t are lump sum taxes net of lump sum non-labour income. The decision for households is somewhat complicated because they have to simultaneously optimise their consumption, C_t , over time and over the range of differentiated goods within each period.

Gali demonstrates that the log-linear optimal labour and consumption decisions from this model can be approximately described by:

$$w_t - p_t = \sigma c_t + \varphi n_t \quad (8)$$

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma} (r_t - E_t \{\pi_{t+1}\} - \rho) \quad (9)$$

where lower case variables are in natural logarithms, $\rho \equiv -\log \beta$ is the discount rate and $r \equiv -\log Q_t$ is the riskless nominal interest rate.

Firms in the NK model are assumed to lie on a continuum indexed $i \in [0,1]$ and produce differentiated goods, $Y(i)$, using identical technology, A_t , and with a production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad (10)$$

Following Calvo (1983), individual firms reset prices optimally in each period with probability $1 - \theta$ which is independent of the time that has elapsed since the firm last changed prices.

From these basic building blocks, Gali shows an approximate log-linear form of the dynamics of aggregate prices can be described as:

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1}) \quad (11)$$

where p_t^* is the optimal price set by firms that reset prices in period t . Equation (11) suggests that inflation in period t in the NK model is due to the deviation between the aggregate price index and the optimal price. The optimal price can then be shown to be:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k|t} - p_{t+k} \} \quad (12)$$

where μ is the log of the desired markup, β is the discount factor for the firms nominal future returns and equal to the discount rate of households, and mc is the real marginal costs of production. Equation (12) suggests that the firm when optimally adjusting prices will choose a desired markup, μ , over a weighted average of marginal costs in the current and future periods where the weights are the probabilities that the price is unchanged over each horizon, θ^k .

Finally, Gali demonstrates that the NK inflation equation is:

$$\hat{\pi}_t = \beta E_t \{ \hat{\pi}_{t+1} \} - \lambda \hat{\mu}_t \quad (13)$$

where $\hat{\cdot}$ indicates the deviation from the steady state values of inflation and the markup, $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \cdot \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$ and the model is solved around a zero steady state rate of inflation. Equation (13) is the NK Phillips curve in inflation-markup space but similar NK Phillips curves can be derived in inflation-output gap and inflation-unemployment rate gap space. Consequently, the defining feature of the NK model is that current inflation depends on the discounted value of expected inflation.

For our purposes there are three aspects to this basic NK Phillips curve to be noted. One, the basic NK curve is solved around the steady state of zero inflation and does not explicitly explain how inflation adjusts to changes in the steady state rates of inflation. Two, β is the discount rate and therefore a parameter inherent to the personal characteristics of the agents in households and firms. And three, the discount rate β is slightly less than 1. If households and firms are risk neutral, face a symmetrical loss function in the region of the optimum price, p^* , and their expectations about future prices are unbiased then assuming a real interest rate of four per cent per annum the quarterly value of β is in the order of 0.99.

2.3 *The Hybrid Phillips Curve*

One of the drawbacks to the NK model noted early on by Fuhrer and Moore (1995), Roberts (1997) and Galí and Gertler (1999) among others is that it implies that reducing inflation is costless if agents are rational and forward looking and that this appears inconsistent with the general observation that anti-inflation policies are associated with large costs to aggregate output. It was also noticed that contrary to the predictions of the NK Phillips curve, lagged inflation plays a significant role in the dynamics of United States inflation. These empirical observations led to the hybrid Phillips curve:

$$\pi_t = (1-\phi)\beta E_t \{\pi_{t+1}\} + \phi\pi_{t-1} - \lambda\hat{\mu}_t \quad 0 < \phi < 1 \quad (14)$$

which is a convex combination of the NK and F-P Phillips curves.¹¹ Note two aspects of the hybrid Phillips curve. One, the curve does not have explicit optimising micro-foundations. And two, in as much as $\beta \approx 1$ in the NK model the dynamic inflation terms sum to one is imposed with no reference to any theory. Instead this constraint is based on prior beliefs handed down from the original F-P and NK theories that the ‘true’ empirical value for ψ in equation (1) is one so that the long-run Phillips curve is vertical.

3. EXPECTATIONS CONSISTENT WITH THE STATISTICAL PROCESS OF INFLATION

The F-P, NK and hybrid theories imply that agents on a fundamental level make systematic errors by believing that the set of all possible statistical processes for inflation is a singleton set containing inflation is an integrated (or near integrated) process. Consider the general expectations operator in equation (1) based on available information. Given the agents in these theories are extremely sophisticated and rich in information as argued above, these same agents for fifty or more years fail to notice that inflation is a bounded variable and therefore not an integrated process.¹² We therefore hypothesise that the set of information available to agents includes knowledge of key aspects of the statistical process of inflation and that the expectations operator should be consistent with that knowledge.

Assuming the general inflation equation (1) is a valid description of the dynamics of inflation we can write:

$$\pi_t = \gamma_0 + \gamma_1 E_{t-1}\{\pi_t\} + \gamma_2 z_t + \zeta_t \quad (15)$$

where inflation in period t is dependent on the expected rate of inflation in period t conditioned on information available in period $t - 1$, a forcing variable z_t and an i.i.d. error

¹¹ Roberts (1997) argues the formation of expectations is partly rational and partly adaptive. The lagged inflation term in the hybrid model can therefore be interpreted as due to a subset of agents who use adaptive expectations when setting prices.

¹² It is ironic that the very great majority of work estimating Phillips curves since Friedman (1968) and Phelps (1967) make use of estimators that are unbiased only if inflation is a stationary process with a constant mean which is in direct conflict with the important prediction of these models that the data should be integrated. More recently when the data is treated as integrated the researchers fail to notice that the inflation is bounded and so unlikely to be ‘truly’ integrated.

term ζ_t . For generality we assume the forcing variable is dependent on lagged values of itself and contemporaneous inflation such that:

$$z_t = \xi_1 z_{t-1} + \xi_2 \pi_t + v_t \quad (16)$$

And therefore:

$$z_t = \frac{\xi_2}{(1-L\xi_1)} \pi_t + \frac{1}{(1-L\xi_1)} v_t \quad (17)$$

where L is the lag operator. Given the simultaneous nature of inflation and the forcing variable this suggests that expected inflation, $E_{t-1}\{\pi_t\}$, depends on the expected values of both inflation and the forcing variable such that:

$$E_{t-1}\{\pi_t\} = E_{t-1}\{\alpha_3 \pi_t + \alpha_4 z_t\} \quad (18)$$

Substituting for z_t in equation (18) using equation (17) we arrive at:

$$E_{t-1}\{\pi_t\} = E_{t-1}\{\xi \pi_t\} \quad (19)$$

where we assume that the expected value of the error term is zero and $\xi = \alpha_3 + \frac{\alpha_4 \beta_2}{(1-L\beta_1)} = 1$ if expectations are unbiased. Equation (19) implies that the expected value of inflation, $E_{t-1}\{\pi_t\}$, can be based on past inflation alone and not on past values of the forcing variable. This means that past values of the forcing variable contain no further information over and above that already incorporated in past values of inflation. In contrast, the inflation equation (15) suggests that the contemporaneous forcing variable contains ‘new’ information which arrives in period t .

Turning now to modelling the expectations of inflation. We can write the expectations operator in equation (15) as:

$$E_{t-1}\{\pi_t\} = f(\bullet) \quad (20)$$

where $f(\bullet)$ describes the dynamic inflation process. Valid functional forms for $f(\bullet)$ depend in part on what we assume agents ‘know’ when forming their expectations of inflation. We

assume that agents know from experience that (i) inflation is a stationary process around a shifting mean, (ii) shocks to inflation are mean zero and stationary; and (iii) other agents are not identical to themselves. The latter implies that agents recognise that the observed mean reversion process of inflation is an aggregation of the disparate adjustment processes of non-identical agents. And consequently, the best an agent can hope for if we eschew the unrealistic assumption of full information is for the agent to understand the average adjustment process of all agents in the economy. Furthermore, without full information agents can only infer the characteristics of the statistical process of inflation from the published aggregate inflation data and not from a detailed understanding of all the non-identical agents in the economy.

The idea that agents only understand the mean reversion process of aggregate measures of inflation suggests that agents only need to be able to approximate that process. Therefore, for simplicity of exposition, assume inflation in continuous time follows an arithmetic Poisson-Gaussian mean reversion process around a shifting mean of the form:

$$d\pi = \tau (\bar{\pi}^m - \pi) dt + \sigma dw \quad (21)$$

where inflation, π , is from a standard normal distribution, $\bar{\pi}^m$ is the long-run mean rate of inflation in regime m which is subject to discrete shifts due to changes in monetary policy, τ is the speed of adjustment back to the mean rate of inflation, σ is the volatility of inflation, $dw = \varepsilon dt^{\frac{1}{2}}$ is an increment of a Wiener process in continuous time and ε is the standard normal distribution.

The shifts in mean are introduced into the inflation process described by equation (21) as:

$$d\bar{\pi}^m = g(*) \quad (22)$$

where $g(*)$ describes the discrete adjustment process of the shifting mean. The size of the mean shift and its associated probability distribution are difficult to estimate by agents and economists alike as the shifts in mean are ‘rare’ events and so there is a lack of data. In our case there appears to be nine shifts in mean over a period of fifty years. One practical approach is to assume the mean reversion process described by equation (21) is independent of the shifts in mean. This implies that the Wiener process, dw , driving inflation back to its

mean is uncorrelated with the discrete shifts in mean in equation (22), $d\bar{\pi}_t$. A defence of the independence assumption is provided below. On a practical level, if we assume independence, we can then estimate the mean reversion process on its own and equation (21) collapses to an Uhlenbeck and Ornstein (1930) (U-O) process also known as the Dixit and Pindyck (1994) model:

$$d\pi = \tau (\bar{\pi} - \pi) dt + \sigma dw \quad (23)$$

The U-O process has a number of useful properties. First, Dixit and Pindyck (1994) demonstrate that equation (23) is the continuous time version of a first order autoregressive process in discrete time where in the limit when $\Delta t \rightarrow 0$ the AR(1) process is:

$$\pi_t - \pi_{t-1} = \bar{\pi} (1 - e^{-\tau}) + (e^{-\tau} - 1)\pi_{t-1} + \epsilon_t \quad (24)$$

where ϵ_t is normally distributed with mean zero and standard deviation σ_ϵ . We can therefore estimate the parameters in equation (24) with a discrete time AR(1) model such that:

$$\pi_t = a + (1 + b) \pi_{t-1} + \epsilon_t \quad (25)$$

where the mean rate of inflation is $\bar{\pi} = -\frac{\tilde{a}}{\tilde{b}}$, the adjustment process $\tau = -\ln(1 + \tilde{b})$, and

the variance $\hat{\sigma} = \hat{\sigma}_\epsilon \sqrt{\frac{\ln(1+\tilde{b})}{(1+\tilde{b})^2 - 1}}$ where σ_ϵ is the standard error from estimating equation (25).

The appropriateness of estimating the mean reversion process independently of the process driving the shifts in mean rests on the validity of assuming the two processes are independent and therefore uncorrelated. Consider the nature of a mean shift and the information contained in lagged values of inflation. Begin by assuming that a regime of N periods has a constant mean rate of inflation. If agents can forecast the shift in mean k periods before the end of the regime then inflation in the last k periods of the regime will begin to adjust to its new mean and therefore have a different mean to the first $N - k$ periods of the same regime. This

contradicts the initial assumption that the inflation regime has a constant mean.¹³ Therefore, in an inflation regime where inflation has a constant mean it is not possible to forecast the impending shift in mean and the assumption that the two processes are independent is valid along with estimating the mean reversion process on its own. This conclusion conforms to our general understanding of structural breaks in that they cannot, or at the very least, are very difficult to forecast by the available information prior to the break.

Return now to the expectations operator in equation (20). Defining the current value of inflation, π_o , and assuming that inflation follows the U-O process described in equation (23) then the expected value of inflation at any future time $t + j$ is:¹⁴

$$E(\pi_{t+j}) = \bar{\pi} + (\pi_o - \bar{\pi})e^{-\tau j} \quad (26)$$

with associated variance:

$$\mathcal{V}(\pi_{t+j} - \bar{\pi}) = \frac{\sigma^2}{2\tau}(1 - e^{-2\tau j}) \quad (27)$$

Note that as j becomes large the expected value of inflation converges on its mean, $\bar{\pi}$, and the variance converges on $\frac{\sigma^2}{2\tau}$.

Finally, substituting for $f(\bullet)$ in equation (20) using equation (25) and for $E_{t-1}\{\pi_t\}$ in equation (15) provides the statistical process consistent (SPC) Phillips curve:

$$\pi_t = \varpi \pi_{t-1} + \bar{\pi}^m + \delta_z z_t \quad (28)$$

where $0 < \varpi < 1$ and expected inflation in period t is believed by agents to follow a mean reverting process around a shifting mean. Assuming i.i.d shocks to inflation the criticism of the F-P model that agents make systematic errors in expected inflation is not observed in

¹³ It may be that the transition between two stable regimes is made up of many small shifts in mean which we cannot identify empirically with available techniques. However, the logic remains that each small shift is not able to be forecasted from information contained in the previous regime.

¹⁴ See appendix 3 for a brief derivation of equations (26) and (27).

equation (28) as inflation differs from the *expected path* of inflation only by mean zero random shocks.

4. A STRUCTURAL BREAK GARCH HYBRID PHILLIPS CURVE

To examine the empirical validity of the three ‘modern’ and the SPC theories of the Phillips curve outlined above we estimate a GARCH hybrid Phillips curve allowing for structural breaks of the form:

$$\pi_t = \delta + \delta_f E_t(\pi_{t+1}) + \delta_b \pi_{t-1} + \delta_\mu \mu_t + \sum_{m=1}^n \varphi_m D_m + \epsilon_t \quad (29)$$

where in the ‘mean’ equation (29) inflation, π_t , depends on expected inflation, $E_t(\pi_{t+1})$, conditioned on information available at time t , lagged inflation, π_{t-1} , a ‘forcing’ variable of the markup, μ_t , shift dummies representing the m inflation ‘regimes’, D_m , and an error term, ϵ_t , due to the random errors of agents and the shocks to inflation. The conditional ‘variance’ equation is specified as a GARCH(1, 1) process:

$$\sigma_t^2 = \omega_0 + \omega_1 \sigma_{t-1}^2 + \alpha \epsilon_{t-1}^2 \quad (30)$$

where the conditional variance, σ_t^2 , is a linear function of a constant, ω_0 , past forecast variances (the GARCH term, σ_{t-1}^2) and past squared residuals from the mean equation (the ARCH term, ϵ_{t-1}^2).¹⁵ The ‘mean’ equation can be estimated asymptotically consistently with two stage least squares (2SLS) but it will not be an efficient estimator when ω_1 and α are non-zero and the model does not account for the heteroscedasticity in the data.

The three ‘modern’ theories of the Phillips curve are nested within the hybrid Phillips curve and can be thought of in terms of restrictions to equations (29). In the F-P Phillips curve $\delta_f = 0$ and $\delta_b = 1$ and agents are purely backward looking. At the other extreme, the NK Phillips Curve of Clarida, Galí and Gertler (1999) and Svensson (2000) imply agents are

¹⁵ The conditional variance is the one-period ahead forecasted variance conditioned, or based, on past information. The GARCH model was introduced simultaneously by Bollerslev (1986) and Taylor (1986). Note that Engle’s (1982) ARCH model is a special case of the GARCH model where $\omega_1 = 0$.

purely forward-looking and $\delta_f = 1-d$ and $\delta_b = 0$ where d is the discount rate. Finally, in the hybrid models of Galí and Gertler (1999) and Galí, Gertler and Lopez-Salido (2001) agents are both backward and forward looking and $\delta_f + \delta_b = 1-d$.

Our SPC Phillips curve set out in Section 3 is also nested within the mean equation (29). If $\delta_f = 0$, $0 < \delta_b < 1$ and $\varphi_m \neq 0$ then the inflation data conforms with the Phillips curve where expectations are consistent with the statistical process of inflation and *inconsistent* with the F-P, NK and hybrid theories of inflation.

4.1 The Data

The model is estimated with quarterly seasonally adjusted United States data for the period March 1960 to December 2010. Inflation is measured as the quarterly change in the natural logarithm of the gross domestic product (GDP) implicit price deflator at factor cost. In keeping with much of the recent NK and hybrid literatures the forcing variable is measured as the natural logarithm of the price series divided by unit labour costs which is equivalent to the inverse of labour's share of national income measured at factor cost.

4.2 Expected Inflation and the Markup

Expected inflation, $E_t(\pi_{t+1})$, is conceptually the forecasted value of inflation based on information available at time t which is assumed to be based on the data published in period $t-1$. The forecasted value of inflation is obtained by regressing inflation on lags of inflation and the markup for periods $t-2$ to $t-5$. For models that include the regime dummies these are also included in the regression. The regression is estimated using ordinary least squares and the static forecast of inflation is included in the estimation of the model with a lead of one period, π_{t+1}^f . To overcome simultaneity bias we also replace the contemporaneous markup term with its static forecast, μ_t^f , from regressing the markup on itself and inflation for periods $t-1$ to $t-4$ as well as the shift dummies in models where they are included. This means that the ordinary least squares estimates of the mean equation (29) are equivalent to 2SLS estimates of the model.

5. ESTIMATING UNITED STATES SHORT-RUN PHILLIPS CURVES

5.1 *The Short-run Phillips Curve*

So as to retrieve the standard results in the literature we begin by estimating the mean equation (29) restricting the parameters on the regime shift dummies and ARCH terms to be zero. Columns 1 and 2 in Table 1 report the hybrid and F-P models respectively which are in general consistent with the standard Phillips curve literature. In column 1 we see that without the breaks in mean and the ARCH effects the sum of the dynamic inflation terms are insignificantly different from 1 and the expected inflation term is larger than lagged inflation which is interpreted in the hybrid Phillips curve literature as forward looking agents dominating backward looking agents in the setting of prices. Note however that the forcing variable is insignificant and that the models suffer from some serial correlation and considerable heteroskedasticity. Similarly when we restrict the lead in inflation to zero the estimates reported in column 2 are in general consistent with the F-P model with the estimated coefficient on the sum of the lagged inflation terms large (0.8079) but significantly less than 1. Again the model suffers from considerable heteroskedasticity.

Columns 3 and 4 of Table 1 report the GARCH(1, 1) hybrid and F-P Phillips curves without the inclusion of shift dummies to account for the shifts in mean inflation. We find that the ARCH components are strongly significant as in the standard ARCH inflation literature. Without accounting for the shifts in mean the ARCH methodology appears to be statistically valid however the estimates of the hybrid Phillips curve are not materially affected from those reported in columns 1 and 2 in the same table.¹⁶

Turning now to the models that incorporate breaks in the mean rate of inflation. Table 2 re-estimates the models incorporating the shift dummies that represent the ten identified inflation regimes. In column 1 we find that accounting for the shifts in mean the lead in inflation is now insignificant. Furthermore, in the F-P model that excludes the lead in inflation the lag in inflation is significantly less than one by a wide margin. Note also that both estimated models continue to suffer from heteroskedasticity.

¹⁶ Modelling the ARCH process increases the efficiency of the estimates but does not alter the expected values of the estimated parameters of the ‘mean’ equation.

Columns 3 and 4 of Table 2 account also for the heteroscedasticity in the data and report the estimates of our full GARCH models with structural breaks. We see that the general estimates of the mean model are largely unaffected but the ARCH and GARCH terms remain significant. Note that with the expected inflation term excluded the single lag in inflation is significant with an estimated value of 0.2431 which continues to be significantly less than one by a wide margin. The final model strongly rejects the restrictions of the F-P, NK and hybrid models and strongly accepts the restrictions consistent with the alternative model of the Phillips curve where expectations are consistent with the statistical process of inflation.

5.2 *The Long-run Phillips Curve*

The structural breaks models reported in Table 2 are effectively estimating ten short-run Phillips curves for the ten mean, or long-run, rates of inflation observed in the data. Given the de-meaned data are stationary it is not surprising that the sum of the dynamic inflation terms are less than one in the models reported in Table 2. This does not mean that the long-run Phillips curve is not vertical. To identify the long-run Phillips curve we need to estimate the long-run value of the markup for each long-run value of inflation which is assumed to equal the mean rate of inflation in each regime. The long-run value of the markup can then be calculated from equation (29) as:

$$\bar{\mu}^m = \frac{1}{\hat{\delta}_u} \left[\bar{\pi}^m (1 - \hat{\delta}_f - \hat{\delta}_b) - \varphi_m \right] \quad (31)$$

where the coefficients are their estimated values from the ‘mean’ equation in Table 2. In our case the long-run value of the forcing variable, $\bar{\mu}^m$, is equivalent to the mean markup in each regime.¹⁷

Assuming the ten combinations of the long-run values of inflation and the markup lie along the long-run Phillips curve we provide two estimates of the long-run curve in Table 3. The first is the linear curve,

¹⁷ Our estimate of the shift dummy in each regime is $\hat{\varphi}_m = \bar{\pi}^m (1 - \hat{\delta}_f - \hat{\delta}_b) - \hat{\delta}_u \bar{\mu}^m$ which when substituted into equation (31) means that the long-run value of the forcing variable in each regime is its mean value.

$$\bar{\pi}^m = \alpha_0 + \alpha_1 \mu^m \quad (32)$$

which reveals there is a significant negative inflation-markup long-run Phillips curve. However, if the long-run Phillips curve is not vertical then it must be non-linear as increases in the mean rate of inflation would eventually violate the lower boundary condition of the definition of the markup. We therefore also report in Table 3 a non-linear long-run Phillips curve:

$$\bar{\pi}^m = \beta_0 \exp(\beta_1 \bar{\mu}^m) \quad (33)$$

in Table 3. Again we find a significant negative non-linear relationship between inflation and the markup.

5.3 *A Visual Representation of the Estimates*

Graph 2 provides a visual representation of the estimates from the GARCH(1,1) structural breaks SPC Phillips curve model (i.e. column 4 in Table 2). The large crosses are the ten combinations of the long-run rates of inflation and the markup. The negative sloping non-linear solid line indicated as LRPC is the estimated exponential long-run Phillips curve from Table 3. The thin negatively sloping lines are the short-run Phillips curves for each of the ten inflation regimes once the inflation dynamics have been exhausted and the ARCH terms are at their mean levels. These short-run curves are drawn for the actual range of the markup for each inflation regime. The actual realisations of inflation and the markup are also shown where the symbols identify which regime the data is drawn from.

From the graph we see the negative slope to both the short-run and long-run Phillips curves. We see that a shock to inflation is initially associated with a large fall in the markup in the short run. With time firms adjust prices and the markup partly recovers. However, the higher mean rate of inflation is associated with a lower mean markup in the long run.

6. ARE THE RESULTS ROBUST?

There are two important dimensions to the robustness of the estimates presented above. First, are the results robust to the plethora of ARCH methodologies that have developed since Engle's (1982) paper? To this end the models were re-estimated using EGARCH, PARCH and IGARCH estimation techniques. It is found that the estimates are not affected in any meaningful way by this range of ARCH type models.

The second dimension is the number of breaks in mean. Some observers might feel uncomfortable about the number of breaks identified in the inflation data and that this is in some way driving the results. However, Perron (1989) demonstrates that if the number of identified breaks in mean is too small then this introduces a positive bias in the estimates of the dynamic inflation terms. Russell *et al.* (2011) demonstrates empirically that the bias due to the unaccounted breaks in mean disproportionately affects the estimated coefficient on the lead of inflation. Consequently, if too few regimes have been identified in the empirical analysis above then this makes it more and not less difficult to obtain the general results reported above.

On the other hand, too many breaks may be incorporated in the analysis above. In particular, one might argue that inflation is a highly persistent process with only one or two breaks in mean and that the large number of breaks employed in the estimation introduces a negative bias to the estimated dynamic terms. Russell *et al.* (2011) demonstrates that as the number of optimally chosen *invalid* breaks increases in the analysis of highly persistent data there is indeed a negative bias to the estimates of persistence. However, the biased estimate of persistence has a lower boundary which is considerably above the estimated persistence reported in Table 2. We can therefore confidently argue that the low estimates of persistence that we identify are not due to the over-breaking of highly persistent data and that the reported estimates are economically meaningful and robust to the number of identified breaks.

7. CONCLUSION

To retrieve the standard empirical results of the F-P, NK and hybrid literatures requires us to impose a very small number of breaks, possibly only two, on the model of inflation when using the last fifty years of United States inflation data. The argument that the 'true' number

of breaks is this small and that there has only been three short-run Phillips curves over the same period is hard to sustain. This is because the entire dialog that supports the existing modern Phillips curve theories considers agents to be very sophisticated and information rich. Consequently, any accommodation by central banks to shocks from whatever source will be quickly identified by these sophisticated agents leading to a shift in the expected, long-run, and therefore mean rates of inflation as well as a corresponding shift in the short-run Phillips curve. Furthermore, if there have only been two structural breaks in the mean rate of United States inflation then this implies that the Federal Reserve Bank of America has accommodated (even a little bit) only two shocks over this fifty year period. Given the difficulties inherent in setting monetary policy with incomplete information this implication would appear difficult to accept. Therefore, at the most fundamental conceptual level the F-P, NK and hybrid theories of the Phillips curve that are populated with very sophisticated agents are incompatible with the argument that there have been very few breaks in the mean rate of inflation over the past fifty years. Alternatively if a more believable larger number of breaks are included in the empirical model so as to conform to the level of sophistication of the agents then these same models are highly inconsistent with the inflation data. In particular, the analysis above suggest that once we account for the shifts in mean inflation there is no significant empirical evidence that the model defining expected rate of inflation in the New Keynesian and hybrid theories plays a significant role in inflation dynamics. Furthermore, the standard interpretation of the Friedman and Phelps Phillips curve is also not supported by the data. In contrast, the United States data is consistent with a form of expectations formation that assumes that agents know the statistical process of inflation, that is, a SPC Phillips Curve.

APPENDIX 1 DATA APPENDIX

The United States data are seasonally adjusted and quarterly for the period March 1960 to December 2010. The United States national accounts data are from the National Income and Product Account tables from the United States of America, Bureau of Economic Analysis. The aggregate data were downloaded via the internet on 27 April 2011. The data are available at www.BillRussell.info.

Variable	Details
Inflation	Nominal gross domestic product (GDP) at factor cost is nominal GDP (Table 1.1.5, line 1) plus subsidies (NIPA Table 1.10, line 10) less taxes (NIPA Table 1.10, line 11). The 'price' series is the GDP implicit price deflator at factor cost calculated as nominal GDP at factor cost divided by constant price GDP at 2005 prices (NIPA Table 1.1.6, line 1). Inflation is the first difference of the natural logarithm of the price series. Note that Graph 1 shows the estimated inflation regimes multiplied by 400 to provide an 'annualised' rate of inflation.
The Markup	Calculated as the natural logarithm of nominal GDP at factor cost divided by compensation of employees paid (NIPA Table 1.10, line 2).

APPENDIX 2 IDENTIFYING THE INFLATION REGIMES

The Bai and Perron (1998) algorithm identifies the dates of k breaks in the inflation series so as to minimise the sum of the squared residuals and thereby identify $k+1$ ‘inflation regimes’. The estimated ‘shifting means’ model is:

$$\pi_t = \gamma_{k+1} + \tau_t \quad (\text{A2.1})$$

where π_t is inflation and γ_{k+1} is a series of $k+1$ constants that estimate the mean rate of inflation in each of $k+1$ inflation regimes and τ_t is a random error. The model is estimated with a minimum regime size (or ‘trimming rate’) of 12 quarters (6 per cent of the total sample) and the final model is chosen using the Bayesian Information Criterion. The model is estimated using quarterly United States inflation data for the period March 1960 to December 2010. The estimated breaks are reported in Table A2. The Bai-Perron technique was estimated using `baiperron.src` and `multiplebreaks.src` programmes written by Tom Doan on RATS 7.2.

Table A2: Estimated Inflation ‘Regimes’ using the Bai-Perron Technique		
<i>Regime</i>	<i>Aggregate Data Dates of the ‘Inflation Regimes’</i>	<i>Mean</i>
1	March 1960 to September 1964	0.003166
2	December 1964 to September 1967	0.007450
3	December 1967 to December 1972	0.011538
4	March 1973 to March 1978	0.018534
5	June 1978 to September 1981	0.021085
6	December 1981 to December 1984	0.010196
7	March 1985 to June 1991	0.007769
8	September 1991 to September 2003	0.004841
9	December 2003 to September 2007	0.008041
10	December 2007 to December 2010	0.003341

APPENDIX 3 DERIVING THE CONDITIONAL MEAN AND VARIANCE OF π_{t+j}

Consider the general function:

$$U_t = e^{\tau t} \pi_t \quad (\text{A3.1})$$

Applying Ito's lemma to equation (A3.1) we get:

$$\begin{aligned} \Rightarrow dU_t &= d(e^{\tau t} \pi_t) = e^{\tau t} d\pi_t + \tau \pi e^{\tau t} dt \\ \Rightarrow e^{\tau t} d\pi_t &= d(e^{\tau t} \pi_t) - \tau \pi e^{\tau t} dt \end{aligned} \quad (\text{A3.2})$$

The Ornstein-Uhlenbeck process for π_t is:

$$d\pi_t = \tau(\bar{\pi} - \pi)dt + \sigma dw_t \quad (\text{A3.3})$$

$$\begin{aligned} \Rightarrow e^{\tau t} d\pi_t &= e^{\tau t} \tau \bar{\pi} dt - e^{\tau t} \tau \pi dt + e^{\tau t} \sigma dw_t \\ \Rightarrow d(e^{\tau t} \pi_t) - \tau \pi e^{\tau t} dt &= e^{\tau t} \tau \bar{\pi} dt - e^{\tau t} \tau \pi dt + e^{\tau t} \sigma dw_t \\ \Rightarrow d(e^{\tau t} \pi_t) &= e^{\tau t} \tau \bar{\pi} dt + e^{\tau t} \sigma dw_t \end{aligned} \quad (\text{A3.4})$$

Taking an integral from time $t = 0$ to t for equation (A3.4) gives:

$$e^{\tau t} \pi_t = \pi_0 + \int_{t=0}^t \tau e^{\tau s} ds + \int_{t=0}^t e^{\tau s} \sigma dw_s \quad (\text{A3.5})$$

And we can write equation (A3.5) in terms of π_t as:

$$\pi_t = \pi_0 e^{-\tau t} + \bar{\pi}(1 - e^{-\tau t}) + \int_{t=0}^t e^{-\tau(t-s)} \sigma dw_s \quad (\text{A3.6})$$

The solution of the stochastic differential equation (A3.6) between s and t , if $0 \leq s \leq t$ and is:

$$\pi_t = \pi_s e^{-\tau(t-s)} + \bar{\pi}(1 - e^{-\tau(t-s)}) + \sigma e^{-\tau t} \int_{t=s}^t e^{\tau u} dw_u \quad (\text{A3.7})$$

The conditional mean and variance of π_{t+j} given π_0 is therefore:

$$E_t[\pi_{t+j}] = \bar{\pi} + (\pi_t - \bar{\pi})e^{-\tau j} \quad (\text{A3.8})$$

$$\text{Var}_t[\pi_{t+j}] = \frac{\sigma^2}{2\tau} (1 - e^{-2\tau j}) \quad (\text{A3.9})$$

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Table 1: United States Phillips Curves – Assuming Constant Mean Inflation

	Two Stage Least Squares		GARCH	
	Hybrid	Friedman-Phelps	Hybrid	Friedman-Phelps
	1	2	3	4
MEAN EQUATION				
π_{t+1}^f	0.6638 (3.8)		0.6792 {4.2}	
π_{t-1}	0.2861 (3.1)	0.4258 (4.4)	0.2872 {2.4}	0.4478 {5.0}
π_{t-2}		0.0698 (0.8)		0.0344 {0.5}
π_{t-3}		0.1328 (2.3)		0.2023 {2.8}
π_{t-4}		0.1795 (2.2)		0.1397 {2.4}
μ_t^f	- 0.0141 (- 0.5)	- 0.0470 (- 2.8)	0.0026 {0.2}	-0.0290 {2.0}
Constant	0.0075 (0.6)	0.0244 (2.9)	-0.0009 {-0.1}	0.0156 {2.1}
Season (June)	-0.0010 (- 2.1)		-0.0012 {-3.5}	-0.0009 {-2.7}
<i>Wald Tests Mean Equation</i>				
$\delta_f + \delta_b$	0.9498	0.8079	0.9663	0.8243]
$\delta_f + \delta_b = 0$	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$\delta_f + \delta_b = 1$	[0.6147]	[0.0043]	[0.6333]	[0.0003]
VARIANCE EQUATION				
ARCH ₁			0.1643 {2.2}	0.1427 {2.3}
GARCH ₁			0.8182 {11.9}	0.8378 {13.9}
Constant			0.0000 {1.0}	0.0000 {1.2}
<i>Wald Tests Variance Equation</i>				
$\sum \alpha_i + \beta_1$			0.9825	0.9805
$\sum \alpha_i + \beta_1 = 1$			[0.6461]	[0.5849]
\bar{R}^2	0.77	0.76	0.76	0.76
<i>Serial Correlation Tests</i>				
LM(1)	[0.1156]	[0.0171]		
LM(1 to 4)	[0.0387]	[0.0460]		
DW	2.11	1.93	2.10	1.95
<i>Heteroscedasticity Tests</i>				
ARCH	[0.0010]	[0.0166]	[0.4732]	[0.5452]
White	[0.0002]	[0.0000]	[0.5872]	[0.6693]
B-P-G	[0.0003]	[0.0002]	[0.4529]	[0.7682]
<i>Information Criteria</i>				
Akaike	-8.7751	-8.7630	-8.9588	-8.9345
Schwarz	-8.6924	-8.6640	-8.8264	-8.7696

**Table 2: United States Phillips Curves
Assuming Breaks in Mean Inflation**

	Two Stage Least Squares		GARCH	
	Hybrid	Friedman-Phelps	Hybrid	Friedman-Phelps
	1	2	3	4
MEAN EQUATION				
π_{t+1}^f	0.1144 (0.7)		0.2366 {1.0}	
π_{t-1}	0.2196 (2.3)	0.2268 (2.4)	0.2311 {3.6}	0.2431 {3.7}
μ_t^f	- 0.0871 (- 2.6)	-0.0960 (-3.5)	-0.0292 {-1.2}	-0.0473 {-2.3}
$D_1 \dots D_{10}$	See Table 2b			
Wald Tests Mean Equation				
$\delta_f + \delta_b$	0.3339	0.2268	0.4677	0.2431
$\delta_f + \delta_b = 0$	[0.0821]	[0.0182]	[0.0575]	[0.0002]
$\delta_f + \delta_b = 1$	[0.0006]	[0.0000]	[0.0309]	[0.0000]
$\varphi_1 \dots \varphi_{10} = 0$	[0.1218]	[0.0000]	[0.5276]	[0.0000]
VARIANCE EQUATION				
ARCH ₁			0.1519 {2.5}	0.1468 {2.4}
GARCH ₁			0.8177 {10.1}	0.8236 {10.4}
Constant			0.0000 {1.2}	0.0000 {1.2}
Wald Tests Variance Equation				
$\sum \alpha_i + \beta_1$			0.9696	0.9704
$\sum \alpha_i + \beta_1 = 1$			[0.5276]	[0.5377]
\bar{R}^2	0.80	0.80	0.80	0.80
Serial Correlation Tests				
LM(1)	[0.1455]	[0.1459]		
LM(1 to 4)	[0.0371]	[0.0800]		
DW	2.08	2.08	2.09	2.10
Heteroscedasticity Tests				
ARCH	[0.0000]	[0.0000]	[0.0366]	[0.0468]
White	[0.0137]	[0.0016]	[0.3653]	[0.2304]
B-P-G	[0.0000]	[0.0000]	[0.2890]	[0.2588]
Information Criteria				
Akaike	-8.9090	-8.9169	-9.0517	-9.0527
Schwarz	-8.6931	-8.7176	-8.7860	-8.8036

Table 2b: United States Phillips Curves – Assuming Breaks in Mean Inflation
Estimated Dummy Variables in the Mean Equation

	Two Stage Least Squares		GARCH	
	Hybrid	Friedman-Phelps	Hybrid	Friedman-Phelps
	1	2	3	4
D_1	0.0454 (2.6)	0.0502 (3.7)	0.0166 {1.3}	0.0263 {2.6}
D_2	0.0479 (2.7)	0.0530 (4.0)	0.0183 {1.4}	0.0288 {2.8}
D_3	0.0469 (2.8)	0.0522 (4.3)	0.0192 {1.5}	0.0300 {3.2}
D_4	0.0517 (2.9)	0.0577 (4.6)	0.0230 {1.6}	0.0354 {3.6}
D_5	0.0547 (3.0)	0.0611 (4.9)	0.0254 {1.7}	0.0385 {3.9}
D_6	0.0483 (2.8)	0.0536 (4.2)	0.0194 {1.4}	0.0303 {3.0}
D_7	0.0474 (2.7)	0.0526 (4.0)	0.0185 {1.4}	0.0290 {2.9}
D_8	0.0458 (2.7)	0.0507 (3.8)	0.0169 {1.3}	0.0269 {2.6}
D_9	0.0493 (2.7)	0.0546 (4.0)	0.0190 {1.4}	0.0298 {2.8}
D_{10}	0.0476 (2.7)	0.0526 (3.7)	0.0171 {1.3}	0.0272 {2.6}

Notes to Tables 1, 2 and 2b

The models are estimated using quarterly data for the period March 1960 to December 2010 using 199 and 200 observations for the hybrid and F-P models respectively. Reported as (), { } and [] are t -statistics, z -statistics and probability values respectively. See Appendices 1 and 2 for details concerning the data and the estimation of the inflation regimes. Two-stage-least-squares estimates are estimated with ordinary least squares with the lead in inflation and the markup replaced by their static ‘forecast’ values (see Section 4.2). Three seasonal dummies were included and eliminated on a ‘5 per cent t -statistic criterion’. 2SLS models estimated with HAC standard errors. ‘Seasonal June’ is a seasonal dummy for the June quarter. GARCH models estimated with maximum likelihood estimator (Marquardt optimising algorithm).

Wald tests report the probability values of the associated F-statistic of the restriction. LM(1) and LM(1 to 4) report the probability values of the Breusch-Godfrey LM test of serially correlated residuals for one lag and one to four lags respectively. ARCH test is the Engle (1982) Lagrange multiplier test for autoregressive conditional heteroskedasticity (ARCH) in the residuals. White tests the null hypothesis of no heteroskedasticity against heteroskedasticity of unknown general form in the residuals (White, 1980). B-P-G is the Breusch-Pagan-Godfrey test which is a Lagrange multiplier test of the null hypothesis of no heteroskedasticity against heteroskedasticity of the form $\sigma_t^2 = \sigma^2 h(z_t' \alpha)$, where z_t is a vector of the independent variables from the mean equation (see Breusch and Pagan, 1979, and Godfrey, 1978). The null hypothesis of the heteroskedasticity tests is no heteroskedasticity. Models estimated with Stata/SE 8.2, Eviews 7.1 and RATS 8.01.

Table 3: Estimates of the Long-run Phillips Curve

Linear:
$$\bar{\pi}^m = 0.1105 - 0.2089 \bar{u}^m, \bar{R}^2 = 0.53$$

(9.4) (-9.0)

The estimated coefficient on \bar{u} is zero is rejected, $\chi^2_1 = 81.6327$, prob-value = 0.0000.
Standard error of the regression: 0.0041.

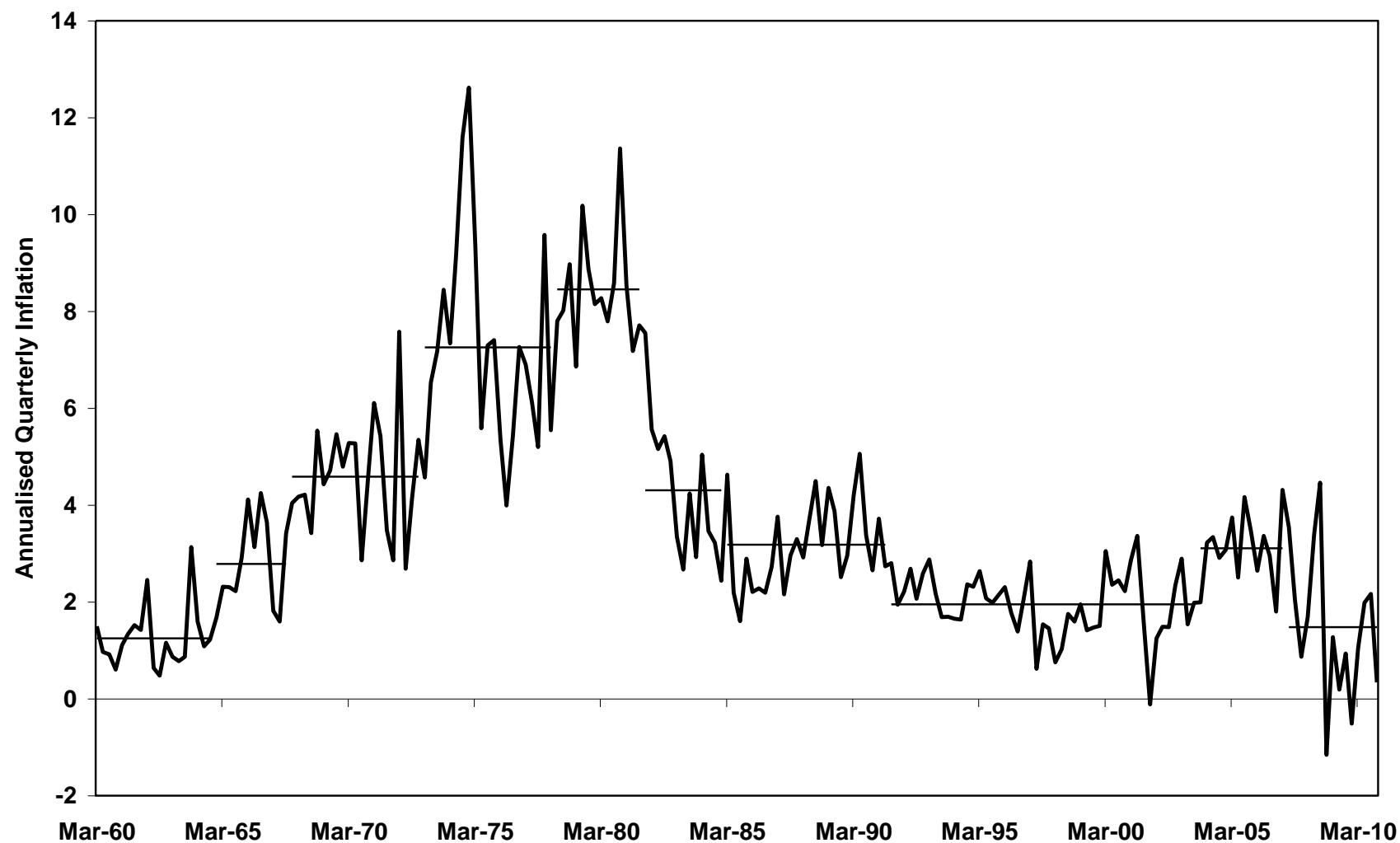
Non-linear Exponential Model
$$\text{Ln}(\bar{\pi}^m) = 6.1276 - 22.6676 \bar{u}^m, \bar{R}^2 = 0.57$$

(7.0) (-13.6)

The estimated coefficient on \bar{u} is zero is rejected, $\chi^2_1 = 184.3145$, prob-value = 0.0000.
Standard error of the regression: 0.4124.

Notes: Numbers in () are t statistics . The models are estimated using ordinary least squares in Eviews 7.1 with Newey-West HAC standard errors. The data are the 10 combinations of the long-run rate of inflation and long-run markup.

Graph 1: United States Annualised Quarterly Inflation, Seasonally Adjusted, March 1960 – December 2010



Notes: Horizontal dashed lines indicate the ten inflation regimes identified by the Bai-Perron technique (see Appendix 2 for details). Annualised quarterly inflation is measured as the change in the natural logarithm of the price index multiplied by 400.

Graph 2: United States Inflation-Markup Phillips Curves
 Quarterly March 1960 to December 2010

